Reliability-based performance evaluation for reinforced railway embankments in the static loading condition

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ABSTRACT: The shift from the allowable stress design method to the limit state design method is advanced now. There is a limit when the soil materials and the reinforcements that have potentially variabilities are quantitatively evaluated though the limit state design method is an effective design method when the performance is designed. In this research, to evaluate the performance of the structure that used the soil materials and the reinforcements quantitatively, the reliability analysis was executed.

1 INTRODUCTIONS

When the safety of the structure is evaluated, it is necessary to handle the soil modulus reasonably and quantitatively though it is known well that the soil modulus vary when various structures are designed intended for the soil and the bedrock. In the design that uses the safety factor in a past allowable stress method, it has corresponded by adopting the value of the safety side as a value for the design when uneven to the ground constant etc. However, it was a problem that the technique for evaluating how taking the value of the safety side logically and quantitatively had not been established.

The reliability design method is a technique for can the evaluation as the safety index or the breakdown probability (Hereafter, it is recorded as the limit state exceedance probability) that is the index to ruin the safety of the structure by applying a probability and a statistical theory to treat the uncertainty logically and quantitatively to the design of the structure and becoming it. Quantitatively treating the soil modulus etc. that have a potential difference becomes possible by applying this technique. Because the limit state exceedance probability is an exceedance probability of defining the state of the limit presented in the performance check type design method, it is not necessarily equivalent to the breakdown probability in case of safety factor <1.

The reliability design method is divided at three levels. A so-called limit state design method hits this in the technique for giving reliability by the load coefficient and the resistance coefficient though the occurrence probability of the mode of breaking is not quantitatively appreciable at level I, Level II is a technique for evaluating reliability requesting the limit state exceedance probability from the safety index obtained from the mean value and the standard deviation of the performance function. Concretely, the First - Order and Second Moment (FOSM) method[2),3),4),5),6)] and the First order reliability (FORM) method[5),6),7),8)], etc. are enumerated. Level III is a technique for requesting the limit state exceedance probability directly from assumption that characteristics of an uncertain factor concerning the mode of breaking the probability statistical are all already-known. Concretely, the Monte Carlo method [5),6),9),10),11)]etc. are enumerated.

The case where the reliability design method is applied to the soil structure is enumerated the plural, and has results especially by the embankment[12),13)] and the having retaining wall[14),15)], etc. in the examination of slope safety[3),4),14),16),17),18),19)]. An analytical research on the reinforcement embankment is actively done recently[20)- 22)].

The reinforcement embankment is targeted in this research. Level II-reliability analytical method in the loading condition was always applied and the height of the embankment or the safety index and the limit state exceedance probability to the performance rank were calculated based on the design basis of the railway[23)]. And, the relation between those indices and conventional safety factor was considered. Refer to As for the calculation of the cost of the life cycle of the reinforcement embankment at the earthquake, it is document 24).

2 ANALYSIS METHOD

2.1 *Calculation of safety factor*

Safety factor FS of the reinforced embankment was calculated by the modified Fellenius method like the next expression.

$$FS = \frac{M_{rw} + M_{rc} + M_{rt} - k_h M_{rk}}{M_{dw} + k_h M_{dk}} \tag{1}$$

K_h is the horizontal seismic coefficient here, M_{dw} is the sliding moments by its own weight, M_{rw} is the resistance moment by its own weight, M_{rc} is the resistant moment by cohesion, M_{rt} is the resistant moment by reinforcements, M_{dk} is the increment of standard sliding moment by unit seismic inertia force, and M_{rk} is the decrease of resistant moment by unit seismic resistant. Because the loading condition is always limited in static this research, the term of the horizontal seismic coefficient is not considered. It based on Moreover, the partial safety coefficient concerning pulling out reinforcement and the partial safety coefficient concerning reinforcement breaking strength were railway standards [23].

2.2 *Calculations of safety index and limit state exceedance probability*

The reliability analysis technique used by this examination is FORM in level II-reliability design method. If the design point is one in FORM, and the performance function is linear, the solution with good accuracy will be obtained in a short time. However, when two or more design points and the performance function is nonlinear, a big error margin is caused. In this examination, the design point used from targeting the embankment constructed on the steady ground and the performance function used FORM with linear by one to be assumable. It easily explains FORM as follows.

The value for the design is assumed, and the function that has X_1 -X_n is assumed to be g, and the value of the performance function is assumed to be Z. The occurrence of the limit state exceedance can be judged as follows.

$$Z = g(X_1, X_2, \cdots, X_n) > 0 \text{ : Safety} \tag{2}$$

$$Z = g(X_1, X_2, \cdots, X_n) \le 0 \text{ : Danger} \tag{3}$$

Here, performance function Z was set as follows.

$$Z = FS - 1 \tag{4}$$

In this research, FS was assumed to be expression (1). If the value of the performance function is a positive value, it can be said excessively of the state of the limit that the structure is safe according to expression (2). If the performance function has 0 or a negative value, it means the state of the limit is exceeded, and there is a structure at risk according to expression (3). It is difficult for performance function Z to become a complex function in case of almost and to request the limit state exceedance probability strictly. Then, the limit state exceedance probability is calculated by performance function's Z in the Taylor expansion, discontinuing the series by the first order term, and making it to linear. In FORM, it expands by the circumference of becoming of the performance function 0. That is, when the Taylor expands by design point $\boldsymbol{x}_j^*$ circumference of a basic random variable, the following expressions obtain performance function Z.

$$Z \approx g\left(x_1^*, x_2^*, \cdots, x_n^*\right) + \sum_{j=1}^{n} \left(X_j - x_j^*\right) \frac{\partial g}{\partial x_j}\bigg|_{x_j^*} \tag{5}$$

Performance function $\boldsymbol{Z}$ mean value μ_z and dispersion σ_z are requested as follows.

$$\mu_Z \approx \sum_{j=1}^{n} \left(\frac{\partial g}{\partial x_j}\bigg|_{x^*} \right) \cdot \left(\mu_{X_j} - x_j^*\right) \tag{6}$$

$$\sigma_Z \approx \sqrt{\sum_{j=1}^{n} \left(\frac{\partial g}{\partial x_j}\bigg|_{x^*} \right)^2 \cdot \sigma_{X_j}{}^2} \tag{7}$$

Safety index β can be obtained as follows by the use of expression (6) and expression (7).

$$\beta = \frac{\mu_Z}{\sigma_Z} \tag{8}$$

A basic random variable is and there is mutually independently a relation between the following up to safety index and limit state exceedance probability P_f(Z ≤0) according to the normal distribution function in case of special.

$$P_f = \Phi(-\beta) \tag{10}$$

Φ is a standard normal probability distribution function, and the safety index β is a standard here by which how mean value Z is relatively away from point (Z = 0) dangerously is shown. That is, the safety index β grows by the mean value of performance function Z large and standard deviation small, and there is room in safety. Moreover, if standard deviation is large even if the mean value of performance function Z is large (equivalence to the safety factor large), it is not necessarily safe. This respect is a big difference point with a past design method.

3 ANALYTICAL MODELS

In this research, the assumption specification described in P.384 of document 23) was examined on the design basis of the railway. There are two kinds of examined structures, that is, reinforced embankment (with long reinforcement) and no reinforced embankment (without long reinforcement). In the assumption specification, the reinforced embankment is specified for the performance rank I and the no reinforced embankment is specified for the performance rank II and III. Figure 1 shows the reinforced embankment model section of the performance rank I, and Figure 2 shows the no reinforced embankment model section of the performance rank II and III.

The height of the embankment was assumed to be four kinds, 3, 4.5, 6, and 9 m. The embankment inclination of the performance rank I was adjusted to 1:1.8 and the performance rank II and III were adjusted to 1:1.5. The embankment section has been divided into two (the surface part and the deep part) according to P.58 of document 23).The range of 2m is called a embankment surface part (henceforth surface) from the embankment slope, and other parts are provided for embankment deep part (henceforth deep)(Fig. 3). The application of the value for the design of the position where reinforcement is constructed and the soil (Table. 1) is directed based on this.

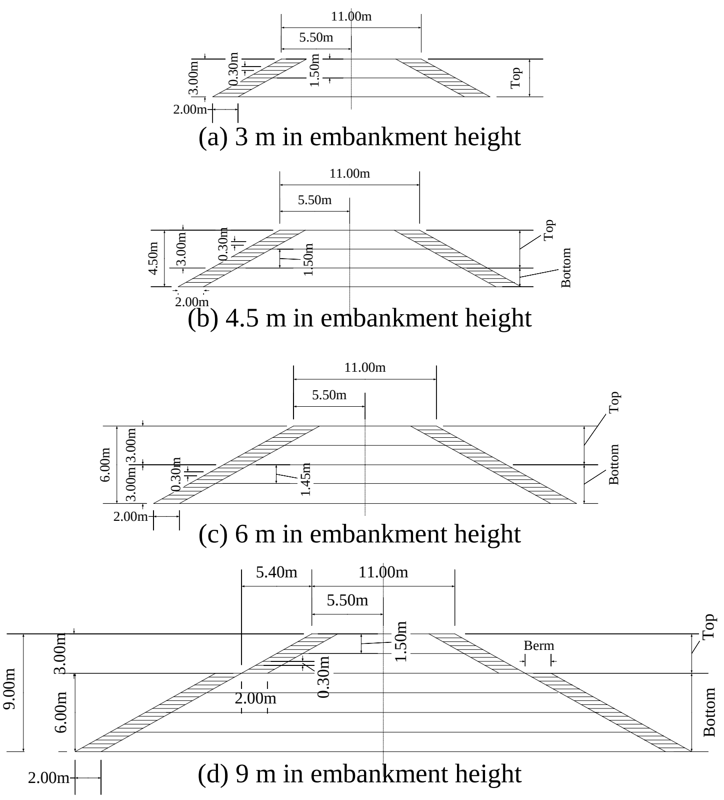
Moreover, there are division into two such as the upper part of embankment (“Top” in figure) and the lower side of embankment (“Bottom” in figure) in the soil.[23)]. Concretely, the whole is treated as the upper part from 3 m in the embankment top.

In case of 4.5 m and 6 m in height, the upper part of the embankment from top to 3 m, and 3 m or less are the lower side of the embankment. In case of 9 m in height, the upper part of the embankment is from top to 3 m, 3 m or less are the lower side of the embankment.

Especially, it was assumed an analytical model by whom berm of 2.0 m in width was installed in the case with 9 m in embankment height because there was regulations that installed berm of 1.5 m in standard width when the height of the embankment exceeded 6 m. That is, The upper part of the embankment and the lower side uses soil 1 for both and the performance rank. The performance rank II uses soil 1 upper part and soil 2 is used lower part. The performance rank III uses soil 2 in the upper part and soil 3 is used lower part .

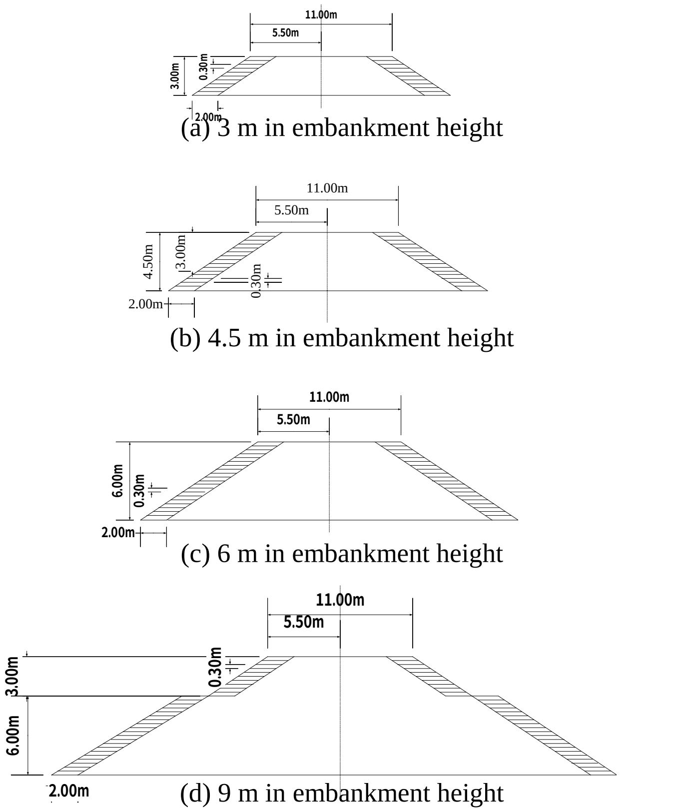
Table.1 Soil material parameters (mean value)

Material		Unit	Soil 1	Soil 2	Soil 3
unit weight γ_t		kN/m^3	18	17	16
Surface part	Cohesion c	kN/m^2	3	3	3
	Internal friction angle Φ	deg.	40	35	30
Deep part	Cohesion c	kN/m^2	6	6	6
	Internal friction angle Φ	deg.	45	40	35



(a) 3 m in embankment height
(b) 4.5 m in embankment height
(c) 6 m in embankment height
(d) 9 m in embankment height

Figure 1. Cross sections of embankments
(Performance rank Ⅰ, gradient of slopes 1:1.8)



(a) 3 m in embankment height
(b) 4.5 m in embankment height
(c) 6 m in embankment height
(d) 9 m in embankment height

Figure 2. Cross sections of embankments
(Performance rank Ⅱ and Ⅲ, gradient of slopes 1:1.5)

Reinforcement was assumed to be two kinds (long and short reinforcement). The construction interval of short reinforcement is 0.3 m and the construction interval of long reinforcement is 1.5 m. The design value of reinforcement is indicated in Table 2. Short reinforcement was constructed in the embankment surface part and long one was constructed in the deep part(Fig. 3). The case of 9 m in height was examined as one example though it became a complex analytical section by the berm installation for 6 m or more in height.

Table 2. Reinforcement tensile strength

Reinforcement	Unit	Mean value
Long	kN/m	30
Short	kN/m	2

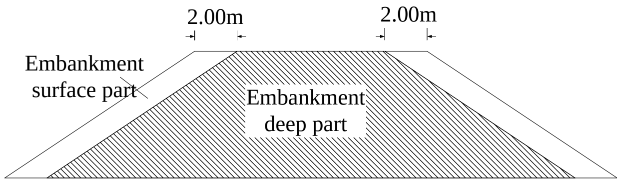
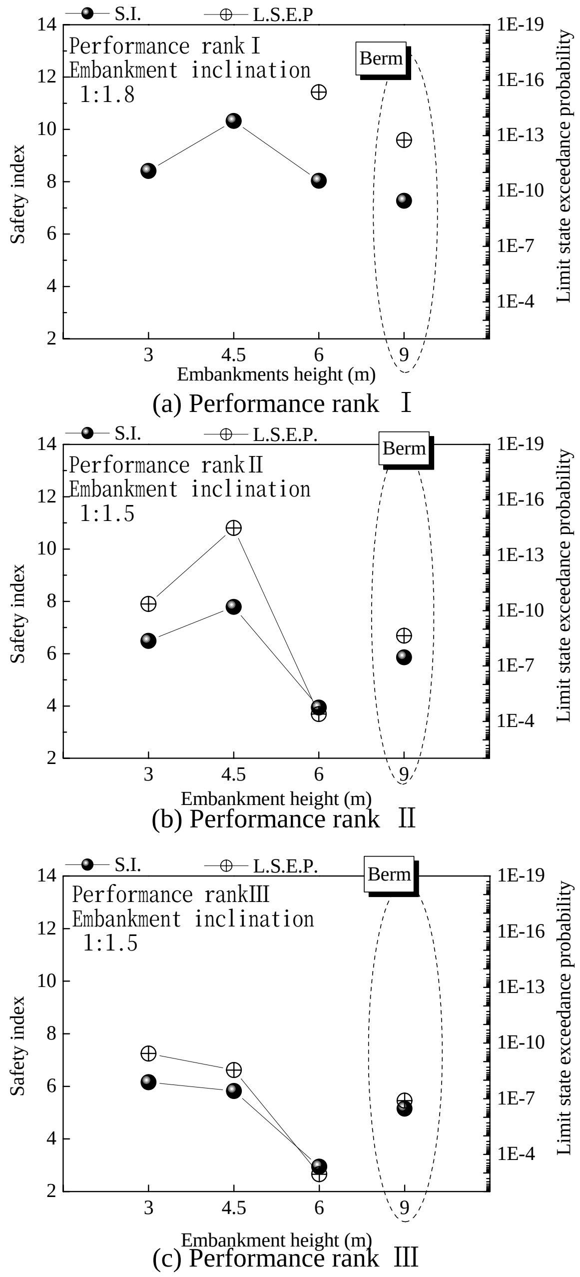


Figure 3. Division of embankment surface and subsurface



(a) Performance rank Ⅰ

(b) Performance rank Ⅱ

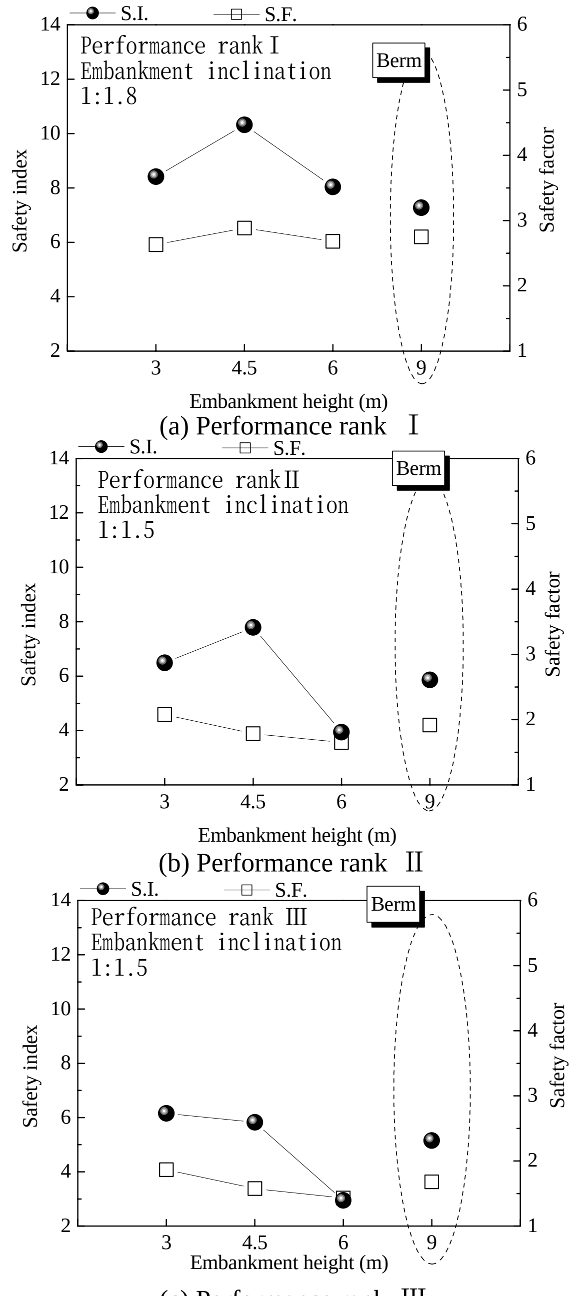
(c) Performance rank Ⅲ

Figure 4. Embankments height versus safety index and limit state exceedance probability

Besides this, the applied loadings[23)] were shown in Table 3, and the applied coefficient of variation[25),26)] is shown in Table 4. Here, the probability distribution of the soil modulus and the reinforcement constant was assumed to be normal distribution and a mutually independent. The mean value of the soil modulus set the value based on the standard of the railway.

Table 3. Overburden loads

Performance rank	Track structure	Unit	Load value
Ⅰ	Concrete	kN/m^2	15
Ⅱ・Ⅲ	Ballast	kN/m^2	10



(a) Performance rank Ⅰ

(b) Performance rank Ⅱ

(c) Performance rank Ⅲ

Figure 5. Embankments height versus Safety index and safety factor

Table 4. Coefficients of variation

Unit weight	0.05
Cohesion c	0.10
Internal friction angle ϕ	0.10
Long reinforcement	0.05
short reinforcement	0.05

4 ANALYSIS RESULTS

Figure 4 shows the safety index (S.I. in figure) and the limit state exceedance probability (L.S.E.P in figure) to the embankment height. Both mutually are equivalent values, and show the safety side so that the larger the value of the safety index is, the smaller the value of the safety side and the limit state exceedance probability is. The calculation result displayed only the range that had been shown in the axis scale. A value that is bigger than prescribed or a small value is not displayed.

As for the safety index, it is understood that the maximum value in 4.5 m in the embankment height is taken in (a), and the value has decreased as the embankment rises, and each height of the embankment is relatively high the value. It is thought that the effect of the reinforcement constructed in embankment deep part is high from this. Moreover, the impetus by the height of the embankment having increased is thought that it is a cause that the resistance power by the reinforcement construction was surpassing as for the reason why the safety index takes the maximum value for 4.5 m in embankment height. In the safety index in (b), the point being seen for the tendency to which the value decreases as similar to (a), the embankment height rises low in (b) is an examination problem in the future. The safety index in (c) is decreased from 3 m in embankment height to 6 m, and has recovered by 9 m. It is thought that the reason why the safety index has recovered in the embankment 9m height is an effect of installing berm. Also, the tendency similar to the safety index is shown about the limit state exceedance probability in each performance rank of (a) - (c).

Figure 5 shows the relation between the safety index and the safety factor (S.F. in figure) to the embankment height. The safety index is the same data as Figure 4. The safety factor is almost constant in (a). As for the safety factor of (b) and (c), the value increases both with an increase of the height of the embankment though it decreases when it reaches 9m. When the safety index is compared with the safety factor, it has been understood that the increase and decrease tendency to safety index and limit state exceedance probability to the height of the embankment is corresponding in (a) - (c). It is thought that the reason why the safety index and safety factor has recovered in the embankment 9 m height is an effect of installing berm. The range of the display of the calculation result is limited as well as Figure 4.

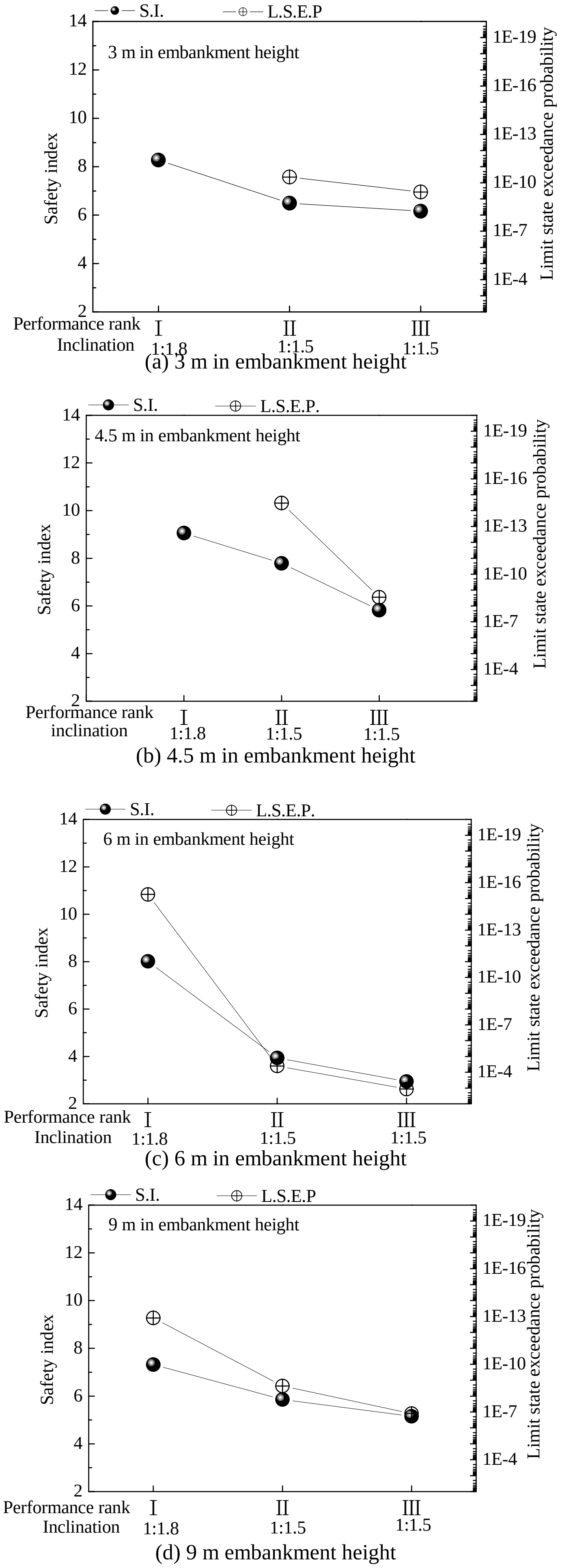
Figure 6 shows the relation between the safety index and the limit state exceedance probability to the performance rank. The embankment inclination of each performance rank was fixed to a horizontal axis. It has been understood that the safety index and the limit state exceedance probability show a similar increase and decrease tendency from (a) to (d).

Figure 7 shows the relation between the safety index and the safety factor to the performance rank. The safety index is the same data as Figure 6. As for the safety factor, the decrease of the value is seen from (a) to (d) in order of performance rank I, II, and III. It has been understood that a similar tendency is seen in the relation between the increase and decrease of the safety index and the safety index and the safety factor.

5 CONCLUSION

The shift from the allowable stress method to the limit state design method is advanced now. There is a limit when the soil materials and the reinforcements that has potentially variabilities are quantitatively evaluated though the limit state design method is an effective design method when the performance is designed. In this research, to evaluate the performance of the structure that used the soil materials and the reinforcements quantitatively, the reliability analysis was executed.

Concretely, the reinforced and no reinforced embankments in the static loading condition of 3, 4.5, 6, 9 m height were modeled that used the design basis of the railway structure standard in Japan. The analysis that assumed circular arc destruction was done, the safety index, the limit state exceedance probability, and the safety factor were calculated. FORM (First order reliability method) was used about the calculation of the safety index and the limit state exceedance probability, and the modified Fellenius method was used for the calculation of the safety factor. FORM has the feature of being obtain the solution with good accuracy in a short time. That is one of the techniques for evaluating reliability by requesting the limit state exceedance probability from the safety index obtained from the mean value and the standard deviation of the performance function. However, it is necessary to meet the requirement, that the design point is the only and the performance function is linear when FORM is applied. Accordingly, embankments constructed on the steady ground where met the requirement was assumed. When modeling, the point to have paid



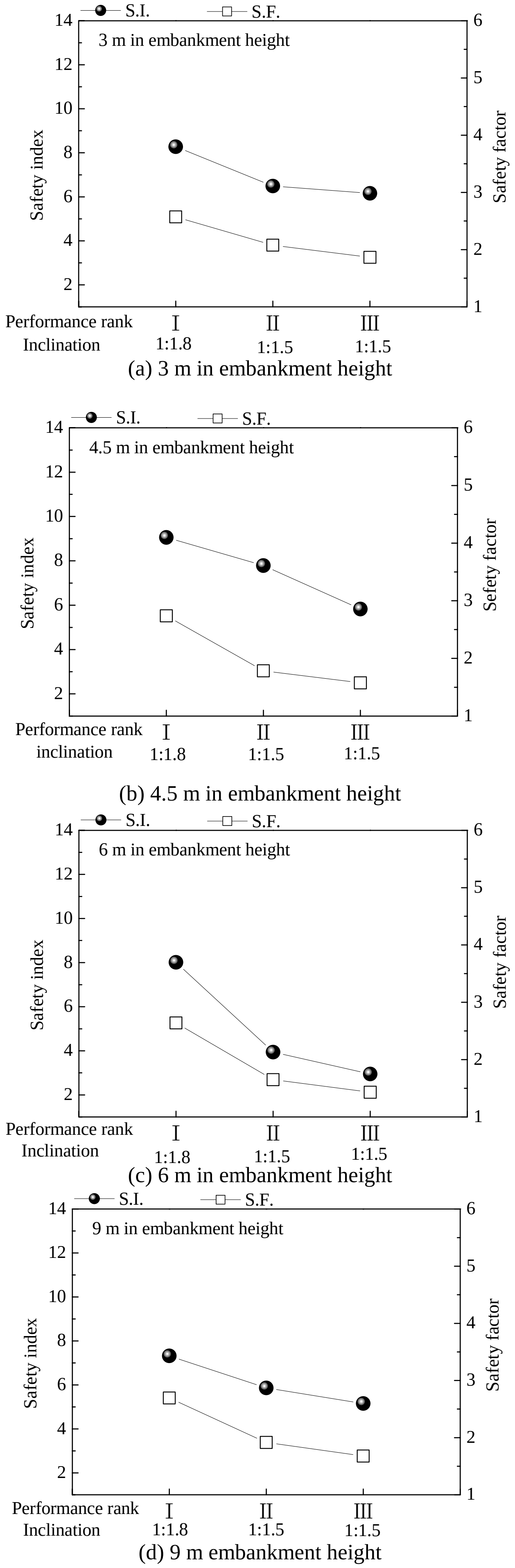
(a) 3 m in embankment height

(b) 4.5 m in embankment height

(c) 6 m in embankment height

(d) 9 m embankment height

Figure 6. Performance rank versus safety index and limit state exceedance probability



(a) 3 m in embankment height

(b) 4.5 m in embankment height

(c) 6 m in embankment height

(d) 9 m embankment height

Figure 7. Performance rank versus safety index and safety factor

attention besides embankments height were grade of face of slopes, arrangement of soil materials, and presence of berm. When the safety index and the limit state exceedance probability that was the result of the reliability analysis and the safety factor that was result of allowable stress method were compared, it was understood that the tendency to safety was approximate corresponding. The feature and the problem in the future of the analytical result are from 1) to 5).

1） In the relation between the embankment height and the safety index, the value becomes small as the embankment rises and the value of the limit state exceedance probability has grown as the embankment rises in 3, 4.5, and 6 m of the performance rank II and III.

2） The relation between the embankment height and the safety factor is constant in the performance rank I, and the value of the safety index has become small as the embankment rises in 3, 4.5, and 6 m of the performance rank II and III.

3） In the relation between the performance rank and the safety index, the safety index becomes small as the performance rank rises. Similarly, in the relation between the performance rank and the limit state exceedance probability, the limit state exceedance probability grows as the performance rank rises.

4） In the relation between the performance rank and the safety factor, the safety factor decreases as the performance rank rises.

5） As for the calculation result of 9 m in the embankment height, with berm, it was shown that stability was comparatively high by arranging the embankment height. It is thought that it is necessary to treat the safety index, the limit state exceedance probability, and the safety factor besides other embankment height, and the examination of analytical model including berm is future tasks.

It was assumed static loading condition in this research. In the future, train loading condition and seismic loading condition, etc. were examined, and the reliability of the reinforcement embankment will be evaluated overall.

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